Wireless Power Transfer: Principles and Prospects

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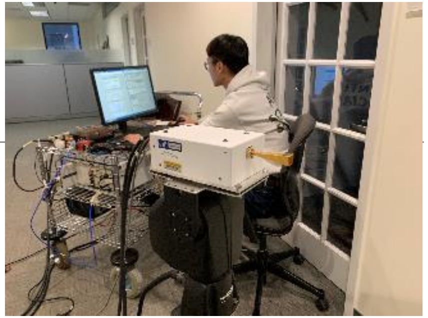
NYU WIRELESS: Mission and Expertise

Leading academic center in wireless communications
- 25 faculty, post-docs, research engineers
- 60 students
- 15 industrial affiliates
- Largest research center in NYU Tandon

Our mission:
- Create future leaders
- Fundamental research: Lead the way to the next generations
- Solve problems for industry

Current in force funding
- Over $10 Million/annually from NSF, NIH, and Corporate sponsors

Theory to Practice



NYU WIRELESS: Technologies and students that impact the real world!

We focus on wireless technologies: “end-to-end”

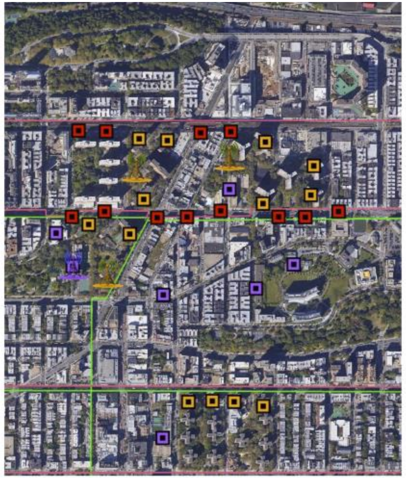
- How wireless interacts with upper layer protocols and applications
- How wireless works in the real world!

NYU WIRELESS tools are widely-used in industry and academia

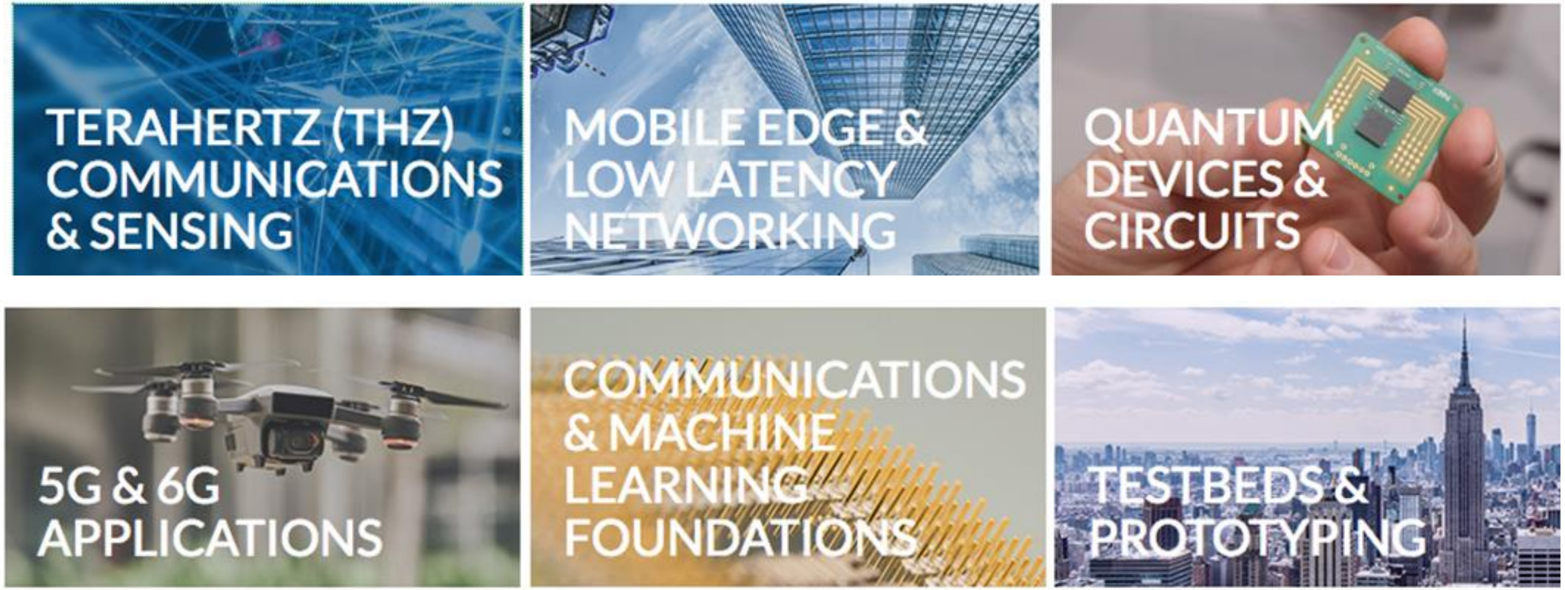
- NYUSIM Statistical Channel Model
- Channel Sounders, Propagation Data, software, chips
- Ns3 network simulator
- Widespread industry and academic use – over 80,000 NYUSIM users

NYU WIRELESS has leading roles in two largest nationwide testbed programs

- NSF PAWR: COSMOS: Large-scale city wide testbed in NYC
- SRC/DARPA: JUMP: Multi-university center on THz



NYU WIRELESS Research Thrusts



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Tom Marzetta – An Introduction

- Born 1951, Washington, D.C.
- Licensed radio amateur, WN(A)3BQK, 1964
- Gonzaga College High School, Washington, D.C., 1964-1968
- Working career
  - Petroleum exploration (Schlumberger-Doll Research, 1978-1987)
  - Defense (Nichols Research, 1987-1995)
  - Telecommunications (Bell Labs [AT&T, Lucent Technologies, Alcatel-Lucent, Nokia], 1995-2017)
  - NYU, 2017-present

Wireless Power Transfer

- Why go after it? Not because it’s easy, but because it’s hard!

- Incalculable potential pay-offs; what if we could wirelessly power:
    - drones?
    - operating room instruments and devices?
    - factory robots?

- We’re still far from realizing this promise!

- There is no known physical principle standing in the way

Classical Beamforming Isn't the Answer

- Transmitted power P_t (watts)
- Power density $\frac{P_t G_t}{4\pi R^2} = \frac{P_t A_t}{\lambda^2 R^2}$ (watts/meter^2)
- Received power $P_r = \frac{P_t A_t A_r}{\lambda^2 R^2}$ (watts)
- Transfer efficiency $\frac{P_r}{P_t} = \frac{A_t A_r}{\lambda^2 R^2} \rightarrow \sqrt{A_t A_r} \approx \lambda R$
- $R = 100$ m, $\sqrt{A_r} = 0.1$ m

Frequency (Ghz)	Wavelength (m)	Transmit aperture (m)
3.0	0.1	100
30	.01	10
300	.001	1

Circuit Theory of Wireless Power Transfer

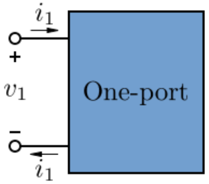
M.T. Ivrlac and J.A. Nossek, "Toward a circuit theory of communication", ***IEEE Trans. Circuits and Systems*****, July 2010**

M.N. Abdallah, T.K. Sarkar, M. Salazar-Palma, "Maximum power transfer versus efficiency", ***IEEE Antennas and Propagation Society International Symposium*****, 2016**

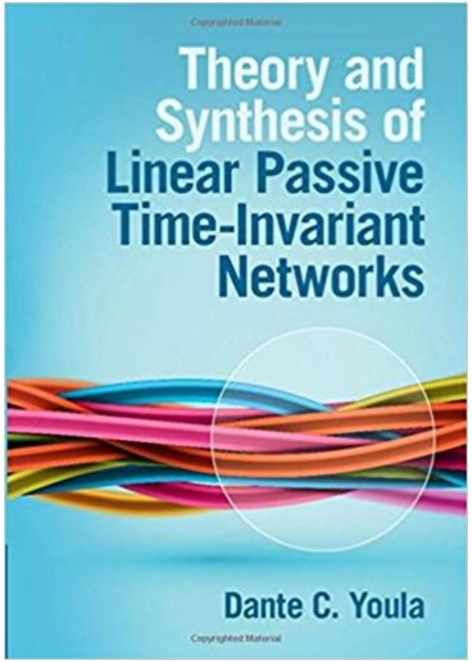
T.L. Marzetta, "Super-directive antenna arrays: Fundamentals and new perspectives", ***Proc. 53nd Asilomar Conference on Signals, Systems, and Computers*****, 2019**

Any System of *n* Transmit/Receive Antennas is an *n*-Port Network

- Ported device



$$v_1(\omega) = z_1(\omega) i_1(\omega)$$



- Linear time-invariant system of *n* ported devices completely described by $n \times n$ complex-valued impedance matrix $\mathbf{Z}(\omega)$

$$v_m(\omega) = \sum_{\ell=1}^{n} z_{m\ell}(\omega) i_\ell(\omega), \quad \mathbf{v}(\omega) = \mathbf{Z}(\omega)\mathbf{i}(\omega)$$

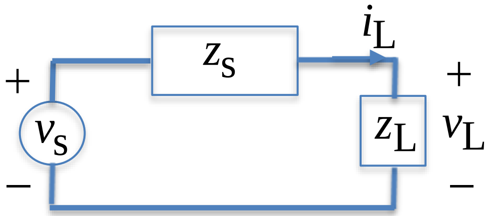
  - reciprocity $z_{m\ell} = z_{\ell m}, \quad \mathbf{Z}^{\mathrm{T}} = \mathbf{Z}$ (*unconjugated* transpose)
  - real power dissipation is non-negative

$$\mathbf{i}^{\mathrm{H}} \operatorname{Re}\{\mathbf{Z}\}\mathbf{i} \geq 0, \forall \mathbf{i}$$

How to Draw Maximum Power From a Voltage Source

- Classical solution: **optimize the load *impedance***

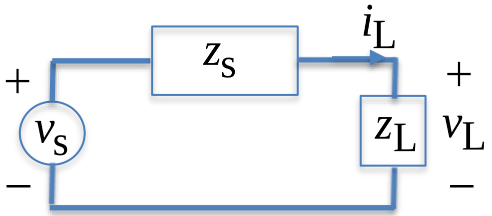
$$P_{\mathrm{L}} = \mathrm{Re}\left\{ i_{\mathrm{L}}^{*} v_{\mathrm{L}} \right\} = \frac{|v_{\mathrm{S}}|^2 z'_{\mathrm{L}}}{\left(z'_{\mathrm{S}} + z'_{\mathrm{L}}\right)^2 + \left(z''_{\mathrm{S}} + z''_{\mathrm{L}}\right)^2} \rightarrow z''_{\mathrm{L}} = -z''_{\mathrm{S}},\ z'_{\mathrm{L}} = z'_{\mathrm{S}}$$



$$P_{\mathrm{L}} = \frac{|v_{\mathrm{S}}|^2}{4 z'_{\mathrm{S}}}$$

- Classical solution: **optimize the load *impedance***

$$P_L = \mathrm{Re}\left\{ i_L^* v_L \right\} = \frac{|v_S|^2 z_L'}{\left(z_S' + z_L'\right)^2 + \left(z_S'' + z_L''\right)^2} \rightarrow z_L'' = -z_S'',\ z_L' = z_S'$$



$$P_L = \frac{|v_S|^2}{4z_S'}$$

- Alternative solution technique: **optimize the load *current* – *a useful trick!***

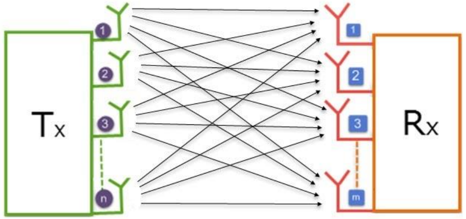
$$P_L = \mathrm{Re}\left\{ i_L^* \left(v_S - i_L z_S\right)\right\} = \mathrm{Re}\left\{ i_L^* v_S \right\} - |i_L|^2 z_S' \rightarrow i_L = \frac{v_S}{2z'}$$

$$\rightarrow P_L = \frac{|v_S|^2}{4z_S'}$$

Only 50% efficient!

Power Transfer Between Two Arrays

$\mathbf{v}_M, \mathbf{i}_M$



Basic Structure of a MIMO System

$\mathbf{v}_N, \mathbf{i}_N$

$$\begin{bmatrix} \mathbf{v}_M \\ \mathbf{v}_N \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{MM} & \mathbf{Z}_{MN} \\ \mathbf{Z}_{MN}^{\mathrm{T}} & \mathbf{Z}_{NN} \end{bmatrix} \begin{bmatrix} \mathbf{i}_M \\ \mathbf{i}_N \end{bmatrix}$$

- Transmit power $P_{\mathrm{T}} = \sum_{m=1}^{M} \mathrm{Re}\left\{ i_{\mathrm{T}m}^{*} v_{\mathrm{T}m} \right\} = \mathrm{Re}\left\{ \mathbf{i}_M^{\mathrm{H}} \mathbf{v}_M \right\} = \mathrm{Re}\left\{ \mathbf{i}_M^{\mathrm{H}} \left[\mathbf{Z}_{MM} \mathbf{i}_M + \mathbf{Z}_{MN} \mathbf{i}_N \right] \right\}$

$$= \mathbf{i}_M^{\mathrm{H}} \mathrm{Re}\left\{ \mathbf{Z}_{MM} \right\} \mathbf{i}_M + \mathrm{Re}\left\{ \mathbf{i}_M^{\mathrm{H}} \mathbf{Z}_{MN} \mathbf{i}_N \right\}$$

- Receive power $P_{\mathrm{R}} = \sum_{n=1}^{N} \mathrm{Re}\left\{ -i_{\mathrm{R}n}^{*} v_{\mathrm{R}n} \right\}$

$$= -\mathbf{i}_N^{\mathrm{H}} \mathrm{Re}\left\{ \mathbf{Z}_{NN} \right\} \mathbf{i}_N - \mathrm{Re}\left\{ \mathbf{i}_N^{\mathrm{H}} \mathbf{Z}_{MN}^{\mathrm{T}} \mathbf{i}_M \right\}$$

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Strategies for Choosing Transmit & Receive Currents

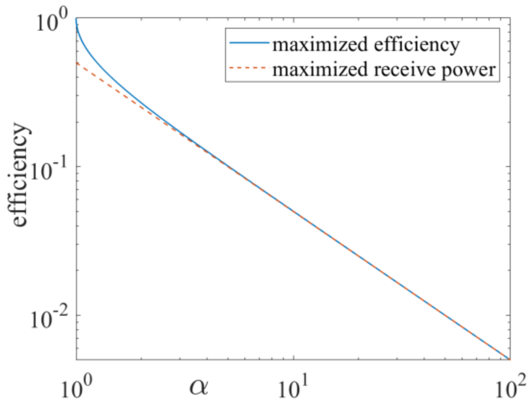
- Power transfer efficiency $$\frac{P_{\mathrm{R}}}{P_{\mathrm{T}}}=\frac{-\mathbf{i}_N^{\mathrm{H}}\,\mathrm{Re}\{\mathbf{Z}_{NN}\}\mathbf{i}_N-\mathrm{Re}\left\{\mathbf{i}_N^{\mathrm{H}}\mathbf{Z}_{MN}^{\mathrm{T}}\mathbf{i}_M\right\}}{\mathbf{i}_M^{\mathrm{H}}\,\mathrm{Re}\{\mathbf{Z}_{MM}\}\mathbf{i}_M+\mathrm{Re}\left\{\mathbf{i}_M^{\mathrm{H}}\mathbf{Z}_{MN}\mathbf{i}_N\right\}}$$
- Greedy strategy (non-cooperative)
  - given the transmit current, choose receive current to maximize receive power:

$$\rightarrow\mathbf{i}_N=-\tfrac{1}{2}\mathbf{Z}_{NN}^{\prime-1}\mathbf{Z}_{MN}^{\mathrm{T}}\mathbf{i}_M\rightarrow\frac{P_{\mathrm{R}}}{P_{\mathrm{T}}}=\frac{\tfrac{1}{4}\mathbf{i}_M^{\mathrm{H}}\mathbf{Z}_{MN}^{*}\,\mathrm{Re}\{\mathbf{Z}_{NN}\}^{-1}\mathbf{Z}_{MN}^{\mathrm{T}}\mathbf{i}_M}{\mathrm{Re}\left\{\mathbf{i}_M^{\mathrm{H}}\left[\mathrm{Re}\{\mathbf{Z}_{MM}\}-\tfrac{1}{2}\mathbf{Z}_{MN}\mathbf{Z}_{NN}^{\prime-1}\mathbf{Z}_{MN}^{\mathrm{T}}\right]\mathbf{i}_M\right\}}$$

  - choose transmit current to maximize resulting efficiency
  - efficiency never exceeds 50%
- Optimum strategy: jointly choose transmit and receiver currents to maximize efficiency
  - optimized transfer efficiency is the same in both directions

Special Case: *Mx1*

- Optimized efficiency $\frac{P_{\text{rec}}}{P_{\text{trans}}} = \alpha - \sqrt{\alpha^2 - 1}, \quad \alpha = \left[1 + \frac{2\left(\mathbf{Z}'_{11} - \mathbf{Z}'^{\text{T}}_{M1}\mathbf{Z}'^{-1}_{MM}\mathbf{Z}'_{M1}\right)}{\mathbf{Z}^{\text{H}}_{M1}\mathbf{Z}'^{-1}_{MM}\mathbf{Z}_{M1}}\right]$
  - 100% efficient if $\left(\mathbf{Z}'_{11} - \mathbf{Z}'^{\text{T}}_{M1}\mathbf{Z}'^{-1}_{MM}\mathbf{Z}'_{M1}\right) = 0$ (perfectly coupled)
- Greedy (non-cooperative) strategy $\frac{P_{\text{rec}}}{P_{\text{trans}}} = \frac{1}{2\alpha}$



Super-Directivity

S.A. Schelkunoff, "A mathematical theory of linear arrays", *Bell Systems Technical Journal*, 1943

G.J. Foschini and M.J. Gans, "On limits of wireless communication in a fading environment when using multiple antennas", *Bell Systems Technical Memorandum*, Sept. 1995

"For example, consider a transmitting horn antenna, with an aperture about 10 wavelengths on a side, located in outer space roughly aimed at the earth, With a one wavelength diameter supergain antenna on the earth it is possible to receive virtually all of the power radiated by the horn antenna."

Super-Directivity: Deliberately Create and Exploit Mutual Coupling; Distinct From Super-Resolution

- Super-resolution
  - Pretends that propagating field is spatially bandlimited (it really isn’t – evanescent waves!)
  - A bandlimited field is analytic: measured field can, in theory, be extrapolated to create a larger, higher-resolution, array (*prolate spheroidal wave functions*)
  - Applicable to synthetic apertures

- Super-directivity
  - Relies on mutual coupling among antennas
  - Can’t be used with synthetic apertures

Example: End-Fire Linear Array

- Impedance matrix

$$\begin{bmatrix} \mathbf{v}_M \\ \mathbf{v}_1 \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{MM} & \mathbf{Z}_{M1} \\ \mathbf{Z}_{M1}^{\mathrm{T}} & \mathbf{Z}_{11} \end{bmatrix} \begin{bmatrix} \mathbf{i}_M \\ \mathbf{i}_1 \end{bmatrix}$$

$$\mathbf{Z}'_{MM} = \begin{bmatrix} 1 & \mathrm{sinc}(kd) & \cdots & \mathrm{sinc}(kd(M-1)) \\ \mathrm{sinc}(kd) & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathrm{sinc}(kd) \\ \mathrm{sinc}(kd(M-1)) & \cdots & \mathrm{sinc}(kd) & 1 \end{bmatrix}, k = \frac{2\pi}{\lambda} \quad \mathbf{Z}_{M1} = \frac{e^{ikz_{\mathrm{R}}}}{z_{\mathrm{R}}} \begin{bmatrix} 1 \\ e^{-ikd} \\ \vdots \\ e^{-ikd(M-1)} \end{bmatrix}$$

- Object is to maximize open-circuit receiver voltage, subject to transmit power constraint

- If we ignore mutual coupling (maximum-ratio; time-reversal beamforming)

$$\text{Max}_{\mathbf{i}_M} |\mathbf{Z}_{M1}^{\mathrm{T}}\mathbf{i}_M|^2, \text{ subject to } \mathbf{i}_M^{\mathrm{H}}\mathbf{i}_M \le P_0 \qquad \mathbf{i}_M = \sqrt{P_0}\frac{\mathbf{Z}_{M1}^*}{\sqrt{\mathbf{Z}_{M1}^{\mathrm{H}}\mathbf{Z}_{M1}}} \rightarrow |v_{\mathrm{R}}|^2 = P_0\mathbf{Z}_{M1}^{\mathrm{H}}\mathbf{Z}_{M1} \propto M$$

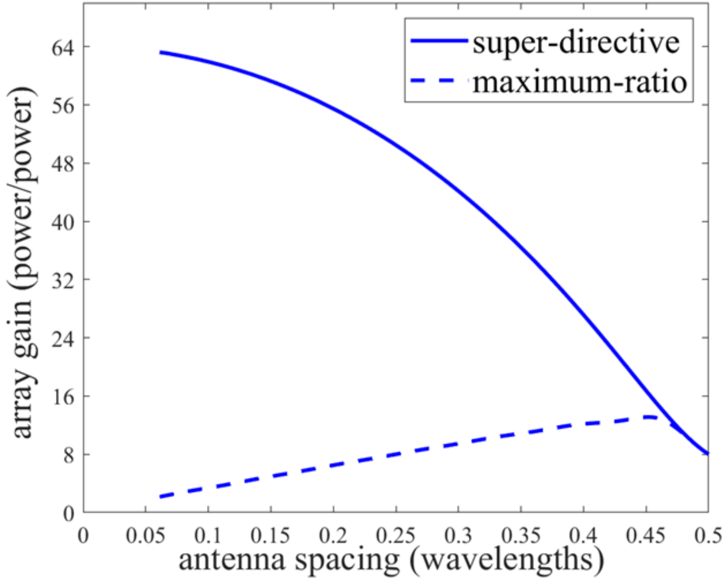
- Super-directive beam-forming

$$\text{Max}_{\mathbf{i}_M} |\mathbf{Z}_{M1}^{\mathrm{T}}\mathbf{i}_M|^2, \text{ subject to } \mathbf{i}_M^{\mathrm{H}}\mathbf{Z}'_{MM}\mathbf{i}_M \le P_0$$

$$\mathbf{i}_M = \frac{\sqrt{P_0}\mathbf{Z}'^{-1}_{MM}\mathbf{Z}_{M1}^*}{\sqrt{\mathbf{Z}_{M1}^{\mathrm{H}}\mathbf{Z}'^{-1}_{MM}\mathbf{Z}_{M1}}} \rightarrow |v_{\mathrm{R}}|^2 = P_0\mathbf{Z}_{M1}^{\mathrm{H}}\mathbf{Z}'^{-1}_{MM}\mathbf{Z}_{M1}\Big|_{d\to 0} \propto M^2$$

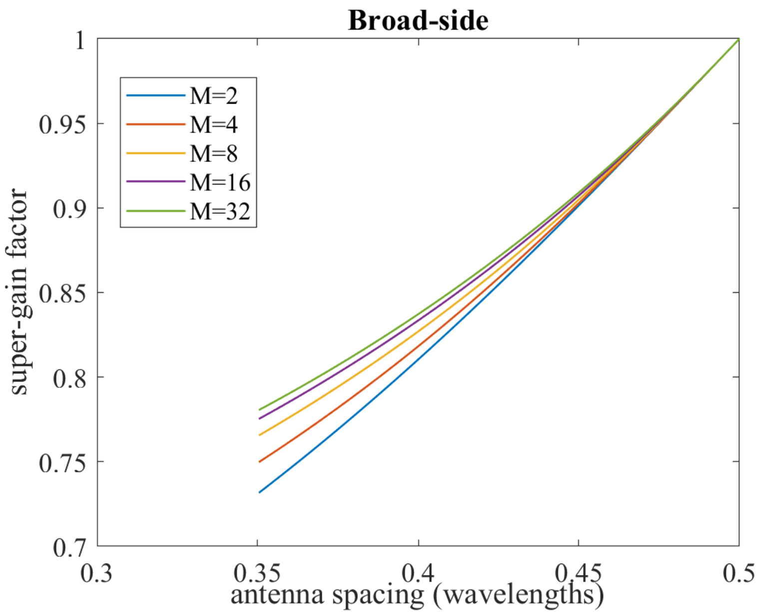
- Super-directivity increases beamforming gain from M to M^2

M=8: Array Gain Relative to Single-Antenna Gain



Broad-Side Linear Array (Receiver Normal to Array Axis): No Super-Directivity; Mutual Coupling Only Makes Things Worse!

Array-Gain/*M*



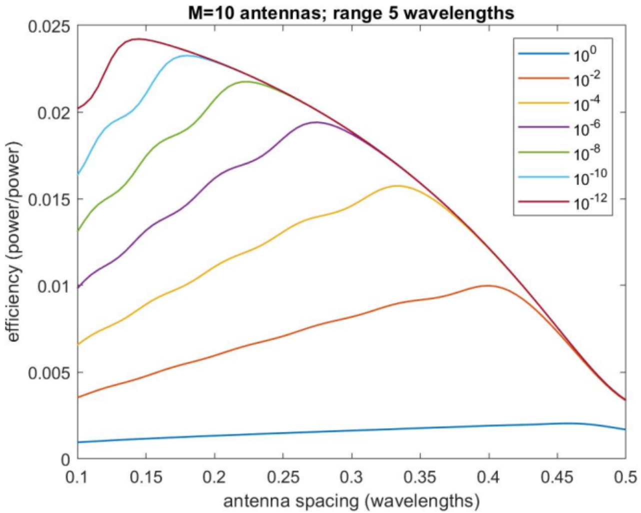
How to Explain Super-Directivity?

- Limit current distribution, *d*→*0*: $i(z) = \sum_{m=0}^{M-1} j_m \frac{d^m \delta(z)}{dz^m}$
    - note: a short dipole is equivalent to a two-element super-directive array!
- Mathematical interpretation: find a sub-space of the real part of the mutual impedance matrix, having small eigenvalues, that contains at least a portion of the propagation vector
    - these modes can be driven by large currents
- Plane-wave expansion of field
    - utilize closely-spaced antennas to create super-wavenumber ($k_z > k$) plane-waves in direction of receiver $k_x^2 + k_y^2 = k^2 - k_z^2 < 0$, so transverse wavenumbers are imaginary
    - transversely, the super-wavenumber plane-waves are evanescent!
        - they decay exponentially and carry only reactive power transversely
    - explains why broad-side operation doesn't support super-directivity

Super-Directive Power Transfer: 10-Antenna Array

- Linear array, end-fire operation, range 5 wavelengths
- Account for antenna internal losses:

  ohmic-resistance/radiation-resistance $\left[10^{0}\ 10^{-2}\ 10^{-4}\ 10^{-6}\ 10^{-8}\ 10^{-10}\ 10^{-12}\right]$



Traditional Problems With Super-Directivity

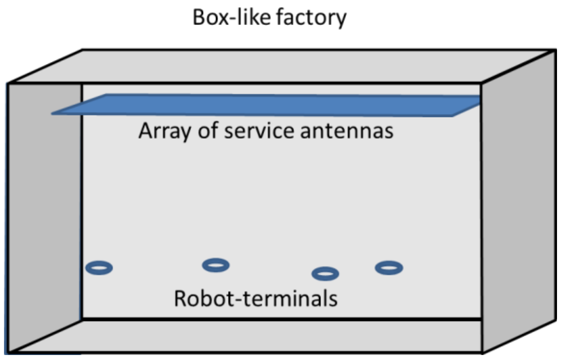
- As antennas get close together, impedance matrix approaches singularity

- Numerical values of antenna currents become enormous
  - real radiated power under control
  - But: reactive power is huge
    - internal ohmic losses
    - huge reactive field in the near-field

- Extreme sensitivity

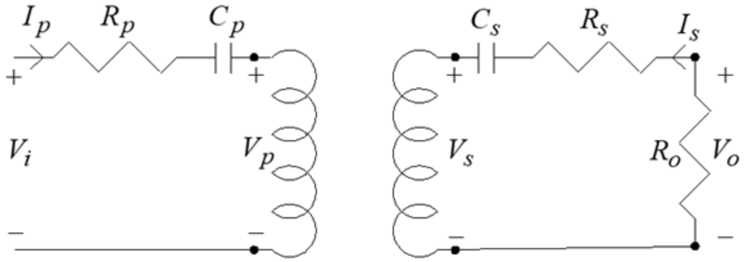
How Can We Make Super-Directivity Practical?

- Super-conducting antennas
- Meta-materials
- New MIMO configurations
- Highly reverberant propagation environment
    - interior isolated from exterior: no spectrum-licensing issues!
    - minimizes near-far effects
    - creates propagation degrees-of-freedom
    - in principle, a single low-gain antenna can transmit arbitrary power to a single low-gain receive antenna with 100% efficiency
- ???



A Big Surprise in 2008: All the Mathematics Had Been in Place For 100 Years, but People Still Didn't Understand its Implications!

A. Karalis, J.D. Joannopoulos, M Soljačić, "Efficient wireless non-radiative mid-range energy transfer", ***Annals of Physics*****, Jan 2008**



$$\frac{P_o}{P_i} = \left(\frac{\kappa \sqrt{Q_p Q_s}}{1 + \sqrt{1 + \kappa^2 Q_p Q_s}} \right)^2$$

- 10 MHz (30-meter wavelength)
- 60 Watts, 40% efficiency
    - high Q's ~ 1000 compensate for low coupling coefficient, $\kappa = .002$

New Graduate Course:

A Linear System Approach to Wave Propagation

Traditional Physicist's Approach to Electromagnetic Theory

- Electric and magnetic fields $\mathbf{E}(t,x,y,z)$ volts/meter, $\mathbf{H}(t,x,y,z)$ amps/meter
- Distributed electric current source $\mathbf{J}(t,x,y,z)$ amps/meter^2
- Maxwell's equations $\nabla\times\mathbf{E} = -\mu\frac{\partial\mathbf{H}}{\partial t}$ $\nabla\times\mathbf{H} = \varepsilon\frac{\partial\mathbf{E}}{\partial t} + \mathbf{J}$ $\varepsilon\nabla\bullet\mathbf{E} = \rho$ $\mu\nabla\bullet\mathbf{H} = 0$
- Potentials $\phi(t,x,y), \mathbf{A}(t,x,y,z): \mathbf{E} = -\nabla\phi - \frac{\partial\mathbf{A}}{\partial t}$ $\mu\mathbf{H} = \nabla\times\mathbf{A}$ $\nabla\bullet\mathbf{A} = -\frac{1}{c^2}\frac{\partial\phi}{\partial t}$
- Uncoupled wave equations $\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi = \rho$ $\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{A} = -\mu\mathbf{J}$
  - solve via method of separation of variables
  - spherical coordinates

Linear System Approach to Electromagnetic Theory

- Maxwell's equations $$\nabla\times\mathbf{E}=-\mu\frac{\partial\mathbf{H}}{\partial t}\quad \nabla\times\mathbf{H}=\varepsilon\frac{\partial\mathbf{E}}{\partial t}+\mathbf{J}\quad \varepsilon\nabla\bullet\mathbf{E}=\rho\quad \mu\nabla\bullet\mathbf{H}=0$$

- A linear space/time-invariant system

$$\overbrace{\begin{bmatrix}\mathbf{E}(t,x,y,z)\\ \mathbf{H}(t,x,y,z)\end{bmatrix}}^{6\times1}=\overbrace{\mathbf{G}(t,x,y,z)}^{6\times3}*\overbrace{\mathbf{J}(t,x,y,z)}^{3\times1}\qquad \begin{bmatrix}\mathbf{E}(\omega,k_x,k_y,k_z)\\ \mathbf{H}(\omega,k_x,k_y,k_z)\end{bmatrix}=\mathbf{G}(\omega,k_x,k_y,k_z)\mathbf{J}(\omega,k_x,k_y,k_z)$$

- Space/time Fourier transforms $$\iiiint dt\,dx\,dy\,dz\left\{\mathbf{Maxwell}(t,x,y,z)\right\}e^{i\omega t}e^{-i(k_x x+k_y y+k_z z)}$$
- System of linear equations $$i\mathbf{k}\times\mathbf{E}=i\omega\mu\mathbf{H}\quad i\mathbf{k}\times\mathbf{H}=-i\omega\varepsilon\mathbf{E}+\mathbf{J}\quad i\varepsilon\mathbf{k}\bullet\mathbf{E}=\rho\quad i\mu\mathbf{k}\bullet\mathbf{H}=0$$

$$\mathbf{ik}\times\mathbf{E}=i\omega\mu\mathbf{H}\quad \mathbf{ik}\times\mathbf{H}=-i\omega\varepsilon\mathbf{E}+\mathbf{J}\quad i\varepsilon\mathbf{k}\bullet\mathbf{E}=\rho\quad i\mu\mathbf{k}\bullet\mathbf{H}=0$$

- Closed-form solution for electric & magnetic fields

$$\begin{bmatrix}\mathbf{E}(\omega,k_x,k_y,k_z)\\ \mathbf{H}(\omega,k_x,k_y,k_z)\end{bmatrix}=\frac{1}{\mathbf{k}^T\mathbf{k}-k^2}\cdot\begin{bmatrix}-\left(\dfrac{k^2\mathbf{I}-\mathbf{k}\mathbf{k}^T}{i\omega\varepsilon}\right)\\ \mathbf{ik}\times\end{bmatrix}\mathbf{J}(\omega,\mathbf{k}),\quad \mathbf{k}^T=\begin{bmatrix}k_x & k_y & k_z\end{bmatrix},\quad k=\frac{\omega}{c}=\frac{2\pi}{\lambda}$$

- Given $\left(\omega,k_x,k_y\right)$, we have a two-pole system in k_z :

$$\mathbf{k}^T\mathbf{k}-k^2=\left(k_z-\sqrt{k^2-k_x^2-k_y^2}\right)\left(k_z+\sqrt{k^2-k_x^2-k_y^2}\right)$$

  - we extract the residues of the two poles
  - fundamental result: in the exterior of the source distribution, the electric and magnetic fields can be represented exactly as superpositions of plane-waves, $\exp\left\{i\left(k_x x+k_y y\pm\sqrt{k^2-k_x^2-k_y^2}\,z\right)\right\}$

Conclusions

- The potential impact of wireless power transfer is enormous
- We need a lot of bold, risky research to make it happen
- Communication theorists and signal processing researchers need to acquire a working knowledge of electromagnetic theory: there is a better way to learn the subject than the physicist's way
- Having a complete mathematical description of a phenomenon does not necessarily mean that we really understand the phenomenon: there is often wide scope for discovery and invention

Simplification of modes of proof is not merely an indication of advance in our knowledge of a subject, but is also the surest guarantee of readiness for further progress.

W. Thomson [1st Baron Kelvin] and P. G. Tait, *Elements of Natural Philosophy*, 1873