# The Mathematical Sciences in 2025
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A National Research Council/National Academies Report for the National Science Foundation

# A Brief History

- Suggested by Tony Chan when he was Assistant Director for MPS at NSF
- Commissioned by Peter March when he was Division Director of DMS
- Study done by the National Research Council/National Academies under the auspices of the Board on Mathematical Sciences and Applications

# The Committee by Field

Core Math: LUIS A. CAFFARELLI, MARK L. GREEN (VC), DAVID EISENBUD, PETER W. JONES (applied also), JU-LEE KIM, JOHN W. MORGAN, YUVAL PERES (also industry)

Applied Math: EMMANUEL J. CANDES (Stat also), PHILLIP COLELLA, MARGARET H. WRIGHT

Statistics: JAMES O. BERGER, JUN LIU (Computational Biology also)

Computer Science: YANN LeCUN, EVA TARDOS

Electrical Engineering: THOMAS E. EVERHART (C)

Finance: TANYA S. BEDER

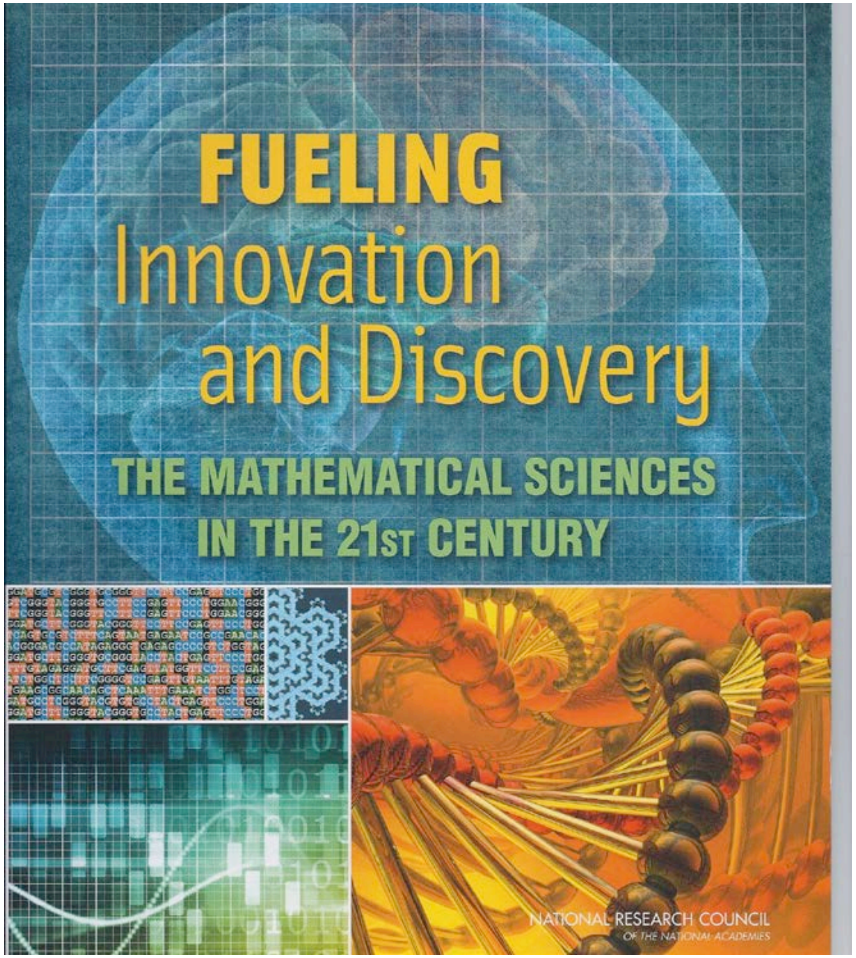
Theoretical Physics: JUAN MALDACENA

Education/Math/CS: JOE B. WYATT

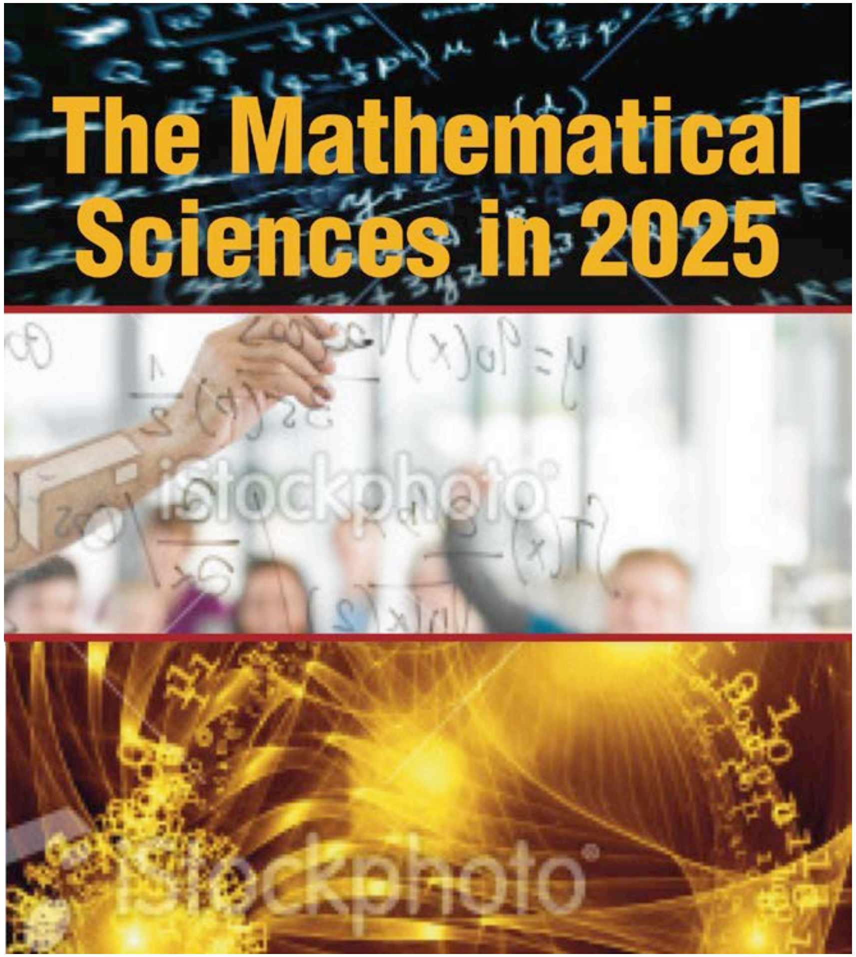
# A Word About Me

- Trained as a pure mathematician
- Directed a start-up NSF institute focused on fostering interactions of mathematics with other fields.  This proved to be a life-changing experience

# Two Publications




The Glossy




The Full Report

# Inside the "Glossy"

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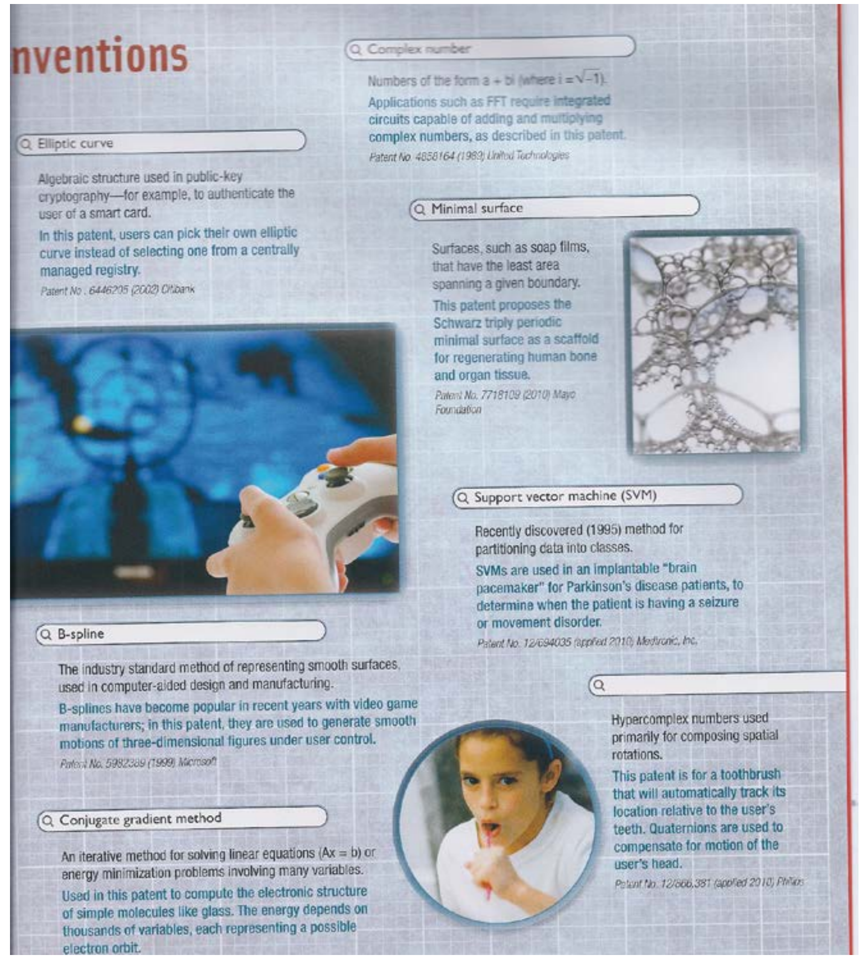

nventions

Complex number

Numbers of the form a + bi (where $i = \sqrt{-1}$).

Applications such as FFT require integrated circuits capable of adding and multiplying complex numbers, as described in this patent.

Patent No. 4858164 (1989) United Technologies

Elliptic curve

Algebraic structure used in public-key cryptography—for example, to authenticate the user of a smart card.

In this patent, users can pick their own elliptic curve instead of selecting one from a centrally managed registry.

Patent No. 6446205 (2002) Citibank

Minimal surface

Surfaces, such as soap films, that have the least area spanning a given boundary.

This patent proposes the Schwarz triply periodic minimal surface as a scaffold for regenerating human bone and organ tissue.

Patent No. 7718109 (2010) Mayo Foundation

Support vector machine (SVM)

Recently discovered (1995) method for partitioning data into classes.

SVMs are used in an implantable "brain pacemaker" for Parkinson's disease patients, to determine when the patient is having a seizure or movement disorder.

Patent No. 12/694035 (applied 2010) Medtronic, Inc.

B-spline

The industry standard method of representing smooth surfaces, used in computer-aided design and manufacturing.

B-splines have become popular in recent years with video game manufacturers; in this patent, they are used to generate smooth motions of three-dimensional figures under user control.

Patent No. 5982389 (1999) Microsoft

Hypercomplex numbers used primarily for composing spatial rotations.

This patent is for a toothbrush that will automatically track its location relative to the user's teeth. Quaternions are used to compensate for motion of the user's head.

Patent No. 12/566,381 (applied 2010) Philips

Conjugate gradient method

An iterative method for solving linear equations (Ax = b) or energy minimization problems involving many variables.

Used in this patent to compute the electronic structure of simple molecules like glass. The energy depends on thousands of variables, each representing a possible electron orbit.

# Chapter Headings
# (Full Report)

- 1. Introduction
- 2. The Vitality of the Mathematical Sciences
- 3. Connections Between the Mathematical Sciences and Other Fields
- 4. Important Trends in the Mathematical Sciences
- 5. People in the Mathematical Sciences Enterprise
- 6. The Changing Academic Context

# Vitality

- Major problems in core mathematics are getting solved, payoff of long-term investment
- Range of applications has dramatically expanded
- New types of mathematics and statistics are being used in applications
- Ubiquity of computation and “big data.”

# Connections with Other Fields

- Enormous expansion of interactions
- Decadal studies of most fields list problems with a major mathematical sciences component
- Role of “dual citizens,” mathematical diaspora
- Need for better mechanisms to connect researchers in other fields with mathematical scientists ; institutes an important mechanism

# Important Trends

- Boundaries within the mathematical sciences are eroding; encourage an inclusive view
- Increasing importance of connections
- Intra-disciplinary collaborations are on the rise
- NSF mathematical sciences institutes play an important role
- Need for a home for computation
- The core is essential, entire ecosystem flourishes together

# People

- Recruitment and retention of women and underrepresented groups should be explicitly included in charge to chairs and vice-chairs; new ideas are needed in recruitment of minorities
- The market for mathematical sciences talent is now global; the US should open its doors as wide as possible to talent from all over the world

# Educating People

- A **major** rethinking of postsecondary mathematical sciences education is needed—content, pathways, delivery
- The mathematical sciences must play a key role in increasing the number of STEM majors and in modernizing their education
- Importance of motivating mathematical ideas by how they are used
- Rising importance of educating those already in the workforce

# The Changing Academic Context

- Cost pressures are severe, and will affect the mathematical sciences disproportionately because of their role in service teaching
- The current business model of mathematical sciences departments is probably unsustainable; faculty levels may drop substantially, preemptive adaptions are needed
- Sustaining research in the mathematical sciences under these conditions requires forethought at NSF

# Broadening the Culture

- Mathematical scientists need a greater awareness of how mathematics is being used
- Reward system needs to evolve, to better appreciate interdisciplinary work and educational innovation
- Growing overlap between core math, applied math, statistics and theoretical CS
- Training should reflect this expanding role

# The Emerging Role of the Mathematical Sciences

**Finding: Mathematical sciences work is becoming an increasingly integral and essential component of a growing array of areas of investigation in biology, medicine, social sciences, business, advanced design, climate, finance, advanced materials, and much more. This work involves the integration of mathematics, statistics, and computation in the broadest sense, and the interplay of these areas with areas of potential application; the mathematical sciences are best conceived of as including all these components. These activities are crucial to economic growth, national competitiveness, and national security. This Finding has ramifications for both the nature and scale of funding of the mathematical sciences and for education in the mathematical sciences.**

# Funding for an Expanded Enterprise

- **Conclusion: The dramatic expansion in the role of the mathematical sciences over the past 15 years has not been matched by a comparable expansion in federal funding, either in the total amount or in the diversity of sources.**

# Funding the Entire Ecosystem

DMS is faced with an innate conflict: As the primary funding unit charged with maintaining the health of the mathematical sciences, it is naturally driven to aid the expansions discussed in Chapter 3 [Connections]; yet it is also the largest of a very few sources whose mission includes supporting the foundations of the discipline, and thus it plays an essential role with respect to those foundations. … There are challenges inherent in supporting a broad, loosely knit community while maintaining its coherence, and the adequacy and balance of funding is a foremost concern. As noted in Chapter 3, funding of excellence wherever it is found should still be the top priority.

# NSF as Enabler

- Within limits, NSF can exercise leadership and serve as an enabler of positive developments. Successful examples include the flourishing Research Experiences for Undergraduates program and NSF's portfolio of mathematical science institutes.
- NSF can, through funding opportunities, enhance the pace of change and facilitate bottom-up developments that capitalize on the energy of members of the community- examples include open calls for workforce proposals, grants to enable the development of new courses and curricula; grants that support interdisciplinary research and research between disciplines within the mathematical sciences, grants that enable individuals to acquire new expertise; and programs that make it easier for young people to acquire experience in industry and to acquire international experience.

# Words from our Chair
# Tom Everhart

“My eyes were opened to the power of the mathematical sciences today, not only as an intellectual undertaking in their own right but also as the increasingly modern foundation for much of science, engineering, medicine, economics, and business...They have demonstrated a great capacity to envision an emerging era in which the mathematical sciences underpin much of twenty-first century science, engineering, medicine, industry, and national security. I hope that this report persuades many others to embrace that vision.”

# Thank You!

# New Majors, New Programs, New Pathways are Needed

Mathematical sciences curricula need attention. The educational offerings of typical departments in the mathematical sciences have not kept pace with the large and rapid changes in how the mathematical sciences are used in science, engineering, medicine, finance, social science, and society at large. This diversification entails a need for new courses, new majors, new programs, and new educational partnerships with those in other disciplines, both inside and outside universities. New educational pathways for training in the mathematical sciences need to be created—for students in mathematical sciences departments, for those pursuing degrees in science, medicine, engineering, business, and social science, and for those already in the workforce needing additional quantitative skills.

# A Deep Rethinking is Needed

**Recommendation: Mathematics and statistics departments, in concert with their university administrations, should engage in a deep rethinking of the different types of students they are attracting and wish to attract, and must identify the top priorities for educating these students. This should be done for bachelor's, master's, and Ph.D.-level curricula. In some cases, this rethinking should be carried out in consultation with faculty from other relevant disciplines.**

# Motivating Mathematical Sciences Ideas by How They are Used

**Recommendation: In order to motivate students and show the full value of the material, it is essential that educators explain to their K-12 and undergraduate students how the mathematical science topics they are teaching are used and the careers that make use of them. Modest steps in this direction could lead to greater success in attracting and retaining students in mathematical sciences courses. Graduate students should be taught about the uses of the mathematical sciences so that they can pass this information along to students when they become faculty members. Mathematical science professional societies and funding agencies should play a role in developing programs to give faculty members the tools to teach in this way.**

# Getting Ahead of the Curve

**Recommendation: Academic departments in mathematics and statistics should begin the process of rethinking and adapting their programs in order to keep pace with the evolving academic environment and to be sure they have a seat at the table as online content and other innovations in the delivery of mathematical science coursework are created. The professional societies have important roles to play in mobilizing the community in these matters, through mechanisms such as opinion articles, online discussion groups, policy monitoring, and conferences.**

# The Global Market for Mathematical Sciences Talent

The market for mathematical sciences talent is now global, and the United States is in danger of losing its global preeminence in the discipline. Other nations are aggressively recruiting U.S.-educated mathematical scientists, especially those who were born in those nations. Whereas for decades the United States has been attracting the best of the world's mathematical scientists, a reverse brain drain is now a real threat. The policy of encouraging the growth of the U.S.-born mathematical sciences talent pool should continue, but it needs to be supplemented by programs to attract and retain mathematical scientists from around the world, beginning in graduate school and continuing through an expedited visa process for those with strong credentials in the mathematical sciences who seek to establish permanent residence.

# Underrepresentation

**Recommendation: Every academic department in the mathematical sciences should explicitly incorporate recruitment and retention of women and underrepresented groups into the responsibilities of the faculty members in charge of the undergraduate program, graduate program, and faculty hiring and promotion. Resources need to be provided to enable departments to monitor and adapt successful recruiting and mentoring programs that have been pioneered at many schools and to find and correct any disincentives that may exist in the department.**

# Toward a Broader Culture

- Conclusion: The mathematical sciences have an exciting opportunity to solidify their role as a linchpin of twenty-first century research and technology while maintaining the strength of the core, which is a vital element of the mathematical sciences ecosystem and essential to its future.
- The enterprise is qualitatively different from the one that prevailed during the latter half of the twentieth century, and a different model is emerging-one of a discipline with a much broader reach and greater potential impact.
- The community is achieving great success within this emerging model, but..

# Toward a Broader Culture (II)

The value of the mathematical sciences to the overall science and engineering enterprise and to the nation would be heightened by increasing the number of mathematical scientists who share the following characteristics:

- They are knowledgeable across a broad range of the discipline, beyond their own area(s) of expertise;
- They communicate well with researchers in other disciplines;
- They understand the role of the mathematical sciences in the wider world of science, engineering, medicine, defense, and business; and
- They have some experience with computation.

It is by no means necessary or even desirable for all mathematical scientists to exhibit these characteristics, but the community should work toward increasing the fraction that does.

# Toward a Broader Culture (III)

The culture within the mathematical sciences should evolve to encourage development of the characteristics listed above.

- The education of future generations of mathematical scientists, and of all who take mathematical sciences coursework as part of their preparation for science, engineering, and teaching careers, should be reassessed in light of the emerging interplay between the mathematical sciences and many other disciplines.
- Institutions, for example, the funding mechanisms and reward systems-should be adjusted to enable cross-disciplinary careers when they are appropriate.
- Expectations and reward systems in academic mathematics and statistics departments should be adjusted so as to encourage a broad view of the mathematical sciences and to reward high-quality work in any of its areas.

# Toward a Broader Culture (IV)

- Mechanisms should be created that help connect researchers outside the mathematical sciences with mathematical scientists who could be appropriate collaborators. Funding agencies and academic departments in the mathematical sciences could play a role in lowering the barriers between researchers and brokering such connections. For academic departments, joint seminars, cross-listing of courses, cross-disciplinary postdoctoral positions, collaboration with other departments in planning courses, and courtesy appointments are useful in moving this process forward.
- Mathematical scientists should be included more often on the panels that design and award interdisciplinary grant programs. Because so much of today's science and engineering builds on advances in the mathematical sciences, the success and even the validity of many projects depends on the early involvement of mathematical scientists.
- Funding for research in the mathematical sciences must keep pace with the opportunities.

# The Core is Essential

Support for basic science is always fragile, and this may be especially true of the core mathematical sciences. In order for the whole mathematical sciences enterprise to flourish long-term, the core must flourish. This requires investment by universities and by the government in the core of the subject. These investments are repaid not immediately and directly in applications but rather over the long term as the subject grows and retains its vitality. From this ever-increasing store of fundamental theoretical knowledge many innovative future applications will be drawn. To give short shrift to maintaining this store would shortchange the country.